

ALGEBRA

List 2.

Polynomials, rational functions, partial fractions

1. Find all integer roots of the following real polynomials:

$$(a) x^3 + x^2 - x + 2; \quad (b) x^4 - 3x^3 + 5x^2 - 9x + 6; \quad (c) x^4 + x^2 - 2.$$

2. Find all rational roots of the following real polynomials:

$$(a) 6x^4 - x^3 + 11x^2 - 2x - 2; \quad (b) x^4 - 5x^2 + 4; \quad (c) 4x^4 + 7x^2 - 2.$$

3. Find all roots of the following real polynomials:

$$(a) x^4 - 6x^2 - 3x + 2; \quad (b) x^4 - 3x^3 - 2x^2 + 2x + 12.$$

4. Find all roots of the following complex polynomials, knowing one of their roots:

$$(a) z^4 + 2z^3 + 4z^2 + 3z + 2, z_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i; \quad (b) z^4 + 3z^3 + 9z^2 + 12z + 10, z_1 = -1 - i.$$

5. Perform the long division and find $Q(x), R(x)$ such that $P(x) = D(x)Q(x) + R(x)$, $\deg(R) < \deg(D)$ for

$$(a) P(x) = x^{12} - 3x^{10} + 2x^7, D(x) = x^3 + 1; \quad (b) P(x) = 2x^8 - 4x^3 + 5x, D(x) = x^2 + x + 1.$$

6. Factor the following real polynomials into irreducible real factors:

$$(a) x^3 - x^2 + x - 1; \quad (b) x^6 + 8; \quad (c) x^4 + 3x^2 + 2.$$

7. Factor the following complex polynomials into irreducible complex factors:

$$(a) z^3 - z^2 + z - 1; \quad (b) z^4 + 3z^2 + 2; \quad (c) z^4 + 1.$$

8. Decompose the following real rational functions into real partial fractions:

$$(a) \frac{x}{(x^2 - 1)(x + 2)}; \quad (b) \frac{x - 1}{x^3 + 1}; \quad (c) \frac{1}{(x^2 - 1)(x + 1)(x - 2)}.$$

9. Decompose the following complex rational functions into complex partial fractions:

$$(a) \frac{1}{z^3 - z^2 + 4z - 4}; \quad (b) \frac{z - 1}{z^3 + 1}; \quad (c) \frac{1}{(z^2 + 2)(z + 1)}.$$